again because of fuselage interference, but also because the problem of unsteady induced drag has not yet been solved in general. Belotserkovskii[21] (pp. 29–33) has considered the Kutta-Joukowsky Theorem "in the small" for unsteady flow. As such, it can only be used for prediction of normal components of aerodynamic loads, and cannot be used to obtain the distribution of the so-called leading edge suction force. The unsteady suction forces are an essential ingredient in the calculation of rotary derivatives. Much work remains to be done, but we cannot agree with the pessimistic evaluation of the state of the art for determining stability derivatives given in Ref. 7, but then—we do not restrict the use of oscillatory aerodynamic theory to flutter analysis.

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Starting Criterion for Hypersonic Inlets

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THE starting criterion for supersonic diffusers, which assumes that the maximum contraction ratio can be calculated solely from normal shock losses, is not valid in some hypersonic applications. This invalidity has been demonstrated in wind-tunnel tests of an early design hypersonic scramjet inlet with a large turbulent boundary layer relative to the inlet height.[1] The purpose of this Note is to present experimental results that show quantitatively when the assumption of only normal shock losses in the starting criteria is adequate.

A one-dimensional adiabatic flow analysis,[2] assuming negligible back pressure, shows that for a supersonic wind tunnel incorporating a second minimum or for a fixed geometry inlet (Fig. 1a) starting is a function of total pressure recovery $p_{t,2}/p_{t,1}$, contraction ratio A_1/A_2, and Mach number M_1. Since the maximum contraction ratio for starting is obtained when the flow at the second minimum is sonic, the previous parameters can be expressed by the following equa-

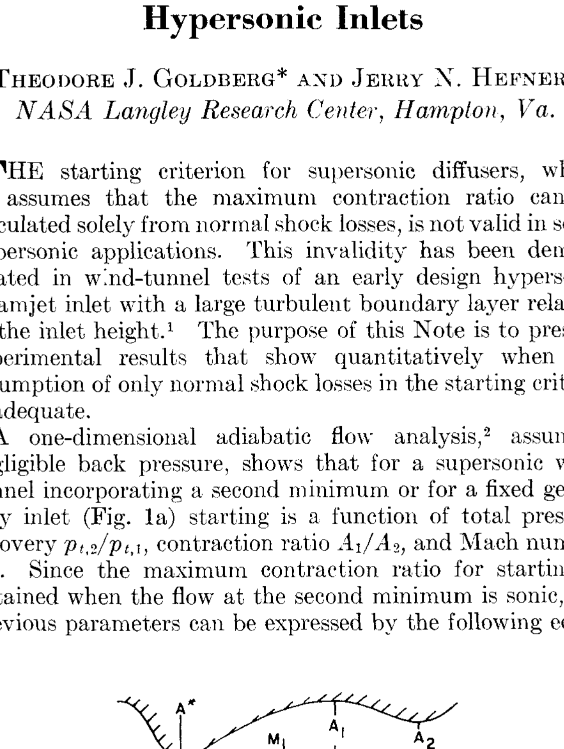

Fig. 1. Starting parameters for hypersonic diffusers.

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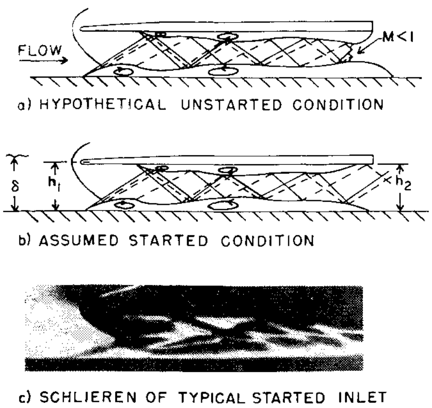

Fig. 2 Flow patterns for inlets with $h_1 \approx \delta$.

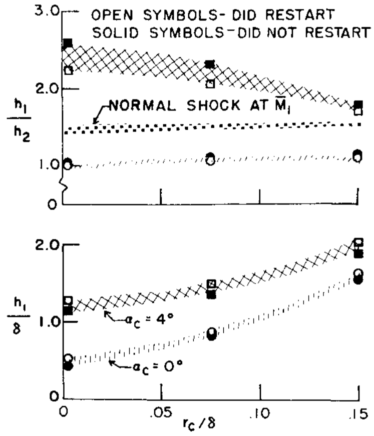

Fig. 4 Starting requirements for two-dimensional inlets: $M_\infty = 6$, $R_\infty/m = 2.2 \times 10^7$, $\delta = 1.016$ cm, $l_c/\delta = 10$.

tion: $A_1/A_2 = (A_1/A^*)(p_{t,2}/p_{t,1})$ and this relationship is shown in Fig. 1b. It should be noted that these required total pressure recoveries are not limited to shock losses. An important point, easily overlooked in inlet design, is that even for a constant area duct ($A_1/A_2 = 1$) the total pressure recovery cannot be below a certain minimum for starting (Fig. 1b).

Since the inlet total pressure recovery governs when the passage will start, an analysis of the starting process requires an examination of the pressure recovery for the condition just prior to the establishment of sonic velocity at the exit or throat. Consider the hypothetical, transient flow model, shown in Fig. 2a, which has subsonic flow just upstream of the exit. This passage could not start if the inlet pressure recovery is less than that required in Fig. 1b. This hypothetical flow pattern cannot be examined experimentally,

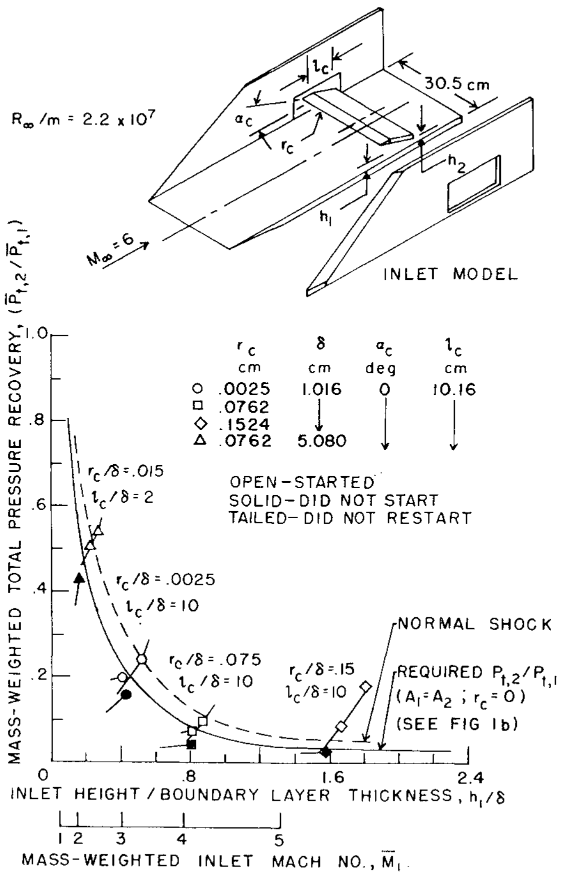

Fig. 3 Effect of relative inlet height on inlet total pressure recovery.

and the complexity of such a flowfield precludes any theoretical analysis at present. However, an exper mental examination can be made for a started passage (Figs. 2b and 2c) for which it can be reasoned that the inlet total pressure recovery would be equal to or greater than that for the assumed transient condition. To illustrate this, a mass-weighted total pressure recovery ($\bar{p}_{t,2}/\bar{p}_{t,1}$) of only 0.08 was measured for a started inlet configuration with parallel surfaces tested in the Langley 20-in. Mach 6 tunnel under near adiabatic wall conditions (see insert of Fig. 3 for model details). Since this is below the total pressure recovery required for starting this configuration, as seen in Fig. 1b, this inlet passage should not be expected to start. In fact, after this passage was unstarted by closing a flap at the exit, the inlet, as predicted, failed to restart upon reopening the flap. (The inlet was initially started by reducing the model wall temperature. The restarted attempt was made at near adiabatic wall conditions.)

Additional experimental results of two-dimensional inlet configurations with turbulent boundary layers (Fig. 3) show that when the mass-weighted total pressure recovery was above that required by the one-dimensional analysis, the inlet did restart (open symbols) after the throat flap was reopened. Also presented in Fig. 3 are results for these inlets, with measured pressure recoveries below those required by the analysis, which did not start (solid symbols), or, after the flap was reopened, did not restart (tailed symbols). Thus, the one-dimensional analysis gives a reasonably good estimate of the inlet total pressure recovery required for starting these inlet configurations.

For inlets with large intake turbulent boundary layers, the usual starting criterion (maximum allowable contraction ratio based on pressure recovery across a normal shock at the inlet face, see Fig. 1b) is invalid because the pressure recovery for started inlets is often below that for the assumed normal shock at the inlet face as seen in Fig. 3. These lower-pressure recoveries are due to losses from many reflected shocks (Fig. 2c) and significant viscous losses. The pressure recovery for this type inlet is sensitive to many design parameters. Although cowl bluntness is the only geometric parameter presented in Fig. 3, cowl length and cowl angle of attack were also investigated and found to significantly effect the inlet pressure recovery. For all configurations investigated, pressure recovery and, therefore, starting were found to be extremely sensitive to small changes in the relative inlet height (h_1/δ). Thus, the relative inlet height must be considered as well as contraction ratio for starting criteria. The use of only contraction ratio can be completely erroneous. For example, Fig. 4 shows that cowl bluntness has essentially no effect on the experimental maximum starting contraction ratio (h_1/h_2) at $\alpha_c = 0°$, but it does have a significant effect on the minimum relative inlet height required for starting. Note that the maximum starting contraction ratios measured for these configurations are considerably below the allowable contraction ratios based on a normal shock at the mass-weighted inlet Mach number ($\bar{M}_1$).

For inlets with predominately inviscid intake flow, the usual concept of contraction ratio based on normal shock losses is an upper limit for the starting criterion. This usual starting criterion also provides conservative guide lines for the starting of mixed compression inlets[3] and internal compression inlets sim lar to the type tested in the present investigation when the inlet height is generally greater than the boundary-layer thickness and the associated pressure recovery is greater than that for a normal shock. This is illustrated in Fig. 4 where experimental y obtained contraction ratios greater than those calculated for normal shock losses are shown for $\alpha_c = 4°$. The measured pressure recoveries for those started configurations were greater than those for a normal shock.

In summary, the inlet total pressure recovery resulting not merely from shock losses but from all shock and viscous losses must be considered in the starting criterion for hypersonic inlets with large turbulent boundary layers relative to the inlet height. The pressure recovery required for starting the inlets of the present investigation was reasonably well predicted by the one-dimensional analysis. The present experiments also show that the inlet pressure recovery is sensitive to many design parameters, particularly the height of the inlet relative to the boundary-layer thickness. When the inlet height is less than the boundary-layer thickness, the usual starting criterion of contraction ratio based on normal shock loss is not adequate for the inlets investigated.

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Method for Predicting Wing Section Pressure Distributions, Lift, and Drag in Transonic Mixed Flow

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THE purpose of this Note is to show how several existing methods for predicting pressure on various parts of an airfoil section in transonic flow can be combined with suitable assumptions to yield the pressure distribution over the entire airfoil, from which the lift and pressure drag (excluding skin-friction drag) can be calculated. The position of the shock wave and the pressure distribution near the leading edge are both calculated. The four methods employed will be outlined only briefly here; the reader is referred to the original publications[1–3] for details.

Sinnott and Osborne[1] have formulated an empirical method for prediction of shock location and pressure distributions on an airfoil from the airfoil crest (where the tangent to the airfoil surface is parallel to the freestream) to the trailing edge. Their method relies heavily on empiricism and on a detailed understanding of the physical phenomena present in a supersonic flow region terminated by a shock wave. The Sinnott and Osborne method was used by the author in toto except that for the present results, compressibility corrections were made using the Kármán-Tsien law instead of the Prandtl-Glauert rule as used by Sinnott and Osborne.†

More recently, Thompson and Wilby[2] have given two methods which between them apply to the calculation of pressure distributions from the airfoil leading edge to the crest. The first of these methods ($TW1$) relies on an empirical relation found between an incompressible velocity distribution near the leading edge stagnation point and the compressible velocity distribution in the same region at a freestream Mach number (M_∞) between 0 and 1. Furthermore, their first formulation is valid only where the surface slope is large and only relates two flows with the same stagnation point.

To apply the $TW1$ procedure, it was necessary to devise a method for predicting the stagnation-point location in a compressible flow. In an incompressible flow, the stagnation point moves rearward as the angle of attack and lift increase. At a constant angle of attack, the stagnation point moves forward and the lift increases as the Mach number increases. In both of these situations, the lift is increasing but the stagnation point moves in opposite directions. These trends seem contradictory, but the following explanation can be offered. We note that the stagnation point for affinely related airfoils moves forward as the thickness ratio decreases and that the total lift of a specified flow system around a body depends on where the stagnation points are confined. On a wing section, the lift is increased by moving the leading edge stagnation point rearward. The similarity rule for subsonic flow tells us that an airfoil of thickness ratio τ in a freestream with Mach number M_∞ is equivalent to an airfoil in incompressible flow with a thickness of $\tau/(1 - M_\infty^2)^{1/2}$. Moreover, the Prandtl-Glauert rule tells us that the lift curve slope of a

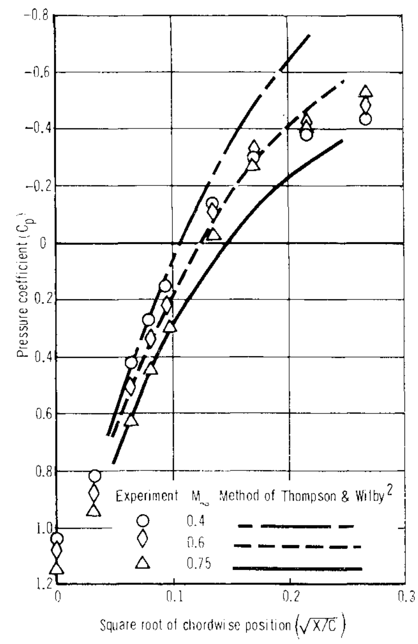

Fig. 1 Comparison of results of method of Thompson and Wilby[2] with experimental data from Ref. 4 NACA 0012 airfoil, zero angle of attack; coordinates for the NACA 0012 can be found in Ref. 6.

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